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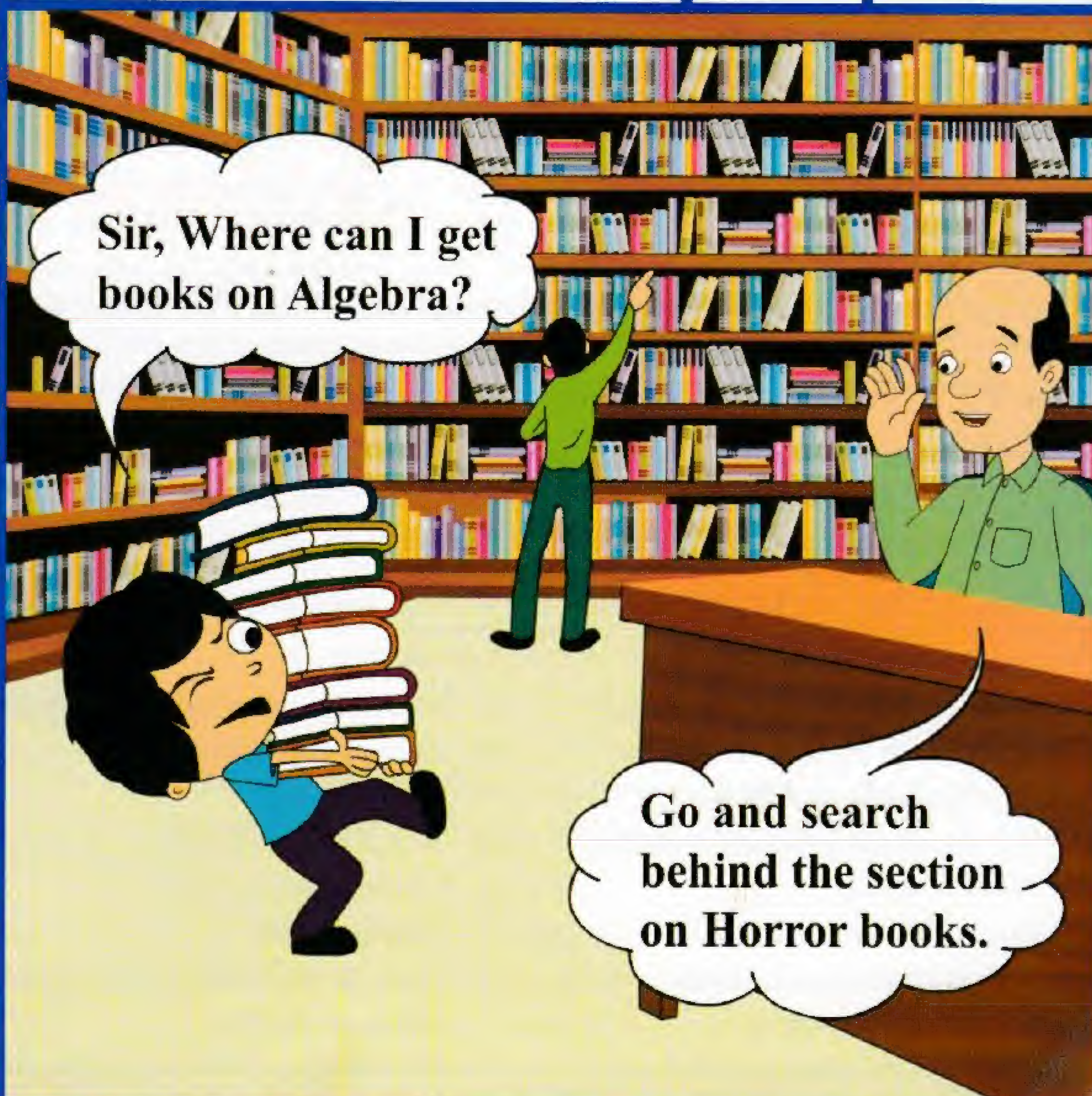

JM

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Junior mathematician (JM) is a Mathematics magazine, principally meant for youngsters of age between 8 and 16.

- Aims to interact directly with the fresh, young and receptive minds, motivating them in their appreciation and application of Mathematics.
- Aspires to present Mathematics as a lovable subject, satisfying to the serious-minded and pleasurable for others, removing math-phobia.
- Provokes young student-authors to write their own discoveries and creative thoughts.

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From the desk of the editor:

MEMORIZATION AND MATHEMATICS

Memorization has been an unpleasant experience in mathematics for many students. When they are in their Primary classes, they are asked to memorize (times) tables. Often people say that memorization is the enemy of critical thinking and conceptual learning; it discourages explorative skills. Learning a mathematical fact through deliberate effort, in isolation from other facts, is not for acquiring knowledge. If you really need to know something, you'll learn it through frequent use; but how do you use something that you don't know? Mathematicians like Professor Jo Boaler say, 'stop with mathematics memorization'. It has become a practice to "value the faster memorizers over those who think slowly, deeply, and creatively," and this has "produced a generation of students who are procedurally competent but cannot think their way out of a box."

Then we have the argument that memorization and automaticity are essential time-saving tools to learn a lot of skills in a short span of time. In real life, children who do not put some direct effort into memorizing their 'times tables' never learn them, and it really holds them back. Many believe that it is really possible for children to memorize the multiplication tables *and* understand what multiplication means. In India, the Vedas, the oldest texts of Hinduism, have been transmitted orally for three thousand years or more. This was possible only through the ability to memorize. What do you do when learning Piano or Carnatic music? A distributed practice system "is helpful in making the procedures second nature, which allows you to focus on the structural elements of the problem." As far as Multiplication tables are concerned, seeing the structure of the table makes the task somewhat easier, because it reminds of the skip-counting pattern. The way this is done provides a visual, auditory, and kinesthetic cue for the student. There have been studies which conclude that "Students, who excel at math, use rote memory to solve simple arithmetic problems, while weak students calculate".

So, how do we evaluate these conflicting views? What do the JMs think about these? Is memorization necessary, evil or both? Please write to us. They can be published, if found suitable.

FRACTIONS &RATIONAL NUMBERS -ARE THEY SAME OR DIFFERENT?

V.Sundaramurthy, formerly of C.P.Jain Hr.Sec.School, Chennai

In one of the recent work-shops of the AMTI, this frequently - discussed question emerged. During debate, each participant expressed his/her own perception but still a majority among them was not comfortable with the explanations.

The reason is simple: most of us have been showing more interest in computations and procedures than in the meaning of the concepts in the subject.

Fractions have certain delicate restrictions and limited role to play in mathematics. You talk about $\frac{1}{4}$ of a cake but never discuss $\frac{-1}{4}$ of it. While expressing fractions, one uses *only* whole numbers {0, 1, 2, ...} as numerators and denominators. This is natural since fractions are precisely related to the concept of 'sharing'.

While dealing with different kinds of numbers and other entities, mathematicians will be interested in the *structure* of the system built with them. For example, they will be interested in properties like closure, associativity, existence of inverse element etc. A neat structure with numbers or other entities lead to development of new ideas that are later used by other disciplines.

Is it possible to get always a natural number as answer to subtraction involving any two natural numbers? When a mathematician finds that Natural numbers are not sufficient for use in certain situations (he 'creates' more numbers such as whole numbers. When whole numbers are themselves seen to be wanting in some areas, Integers are built. When sharing or dividing are involved, fractions are constructed.

Think of this situation: If you have only Integers and Fractions and no Rational numbers. You can think of $\frac{1}{4}$ or $\frac{8}{4}$ but not $\frac{-1}{4}$. (Is $\frac{-1}{4}$ same as $-\frac{1}{4}$?) Fractions are *not* nice enough to visualize/explain how -1 could be divided by 4. Thus arises the need for a new variety of numbers that could meaningfully allow 'fractions' like $\frac{-1}{4}$.

Consider the quotient of any two integers, say, for example, -6 divided by +2. The result -3 is also an integer. That is a nice thing to happen. But, if you find the quotient when +2 is divided by -6, is the result an integer? No. We say that integers are *not closed* with respect to division; it means that the result is not a member of the set of integers. For ensuring '*closure*' , mathematicians introduce another collection of numbers: "Rational numbers".

A rational number is any number that can be expressed as a quotient or fraction $\frac{p}{q}$ of two integers, a numerator p and a non-zero denominator q. (Every integer is also a rational number since q can be 1). A number which is not rational is called an *irrational* number.

With the introduction of this variety, the problem of 'getting the quotient of two integers as an integer' is solved. Now the non-zero Rational are closed with respect to division also.

The need to 'create' Rational numbers can be seen from another angle also. Suppose you have only integers and fractions. In such a situation 'subtraction' is *not* a *closed* operation. For example, one cannot find the answer to sums such as

$$-5 - (\tfrac{2}{3}).$$

This, again, persuades us to introduce Rational numbers; one is also tend to believe that rational numbers differ from fractions in some respects.

- A rational number is a fraction of integers.
- Non-zero Rational numbers are closed with respect to subtraction. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers $\frac{a}{b} - \frac{c}{d}$ is also a rational number.
- Rational numbers are closed with respect to division $\frac{ad}{bc}$. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers $\frac{a/b}{c/d}$ (which is same as $\frac{ad}{bc}$) is also a rational number.
- Not all fractions are rational. For example a fraction with an irrational number in the numerator or denominator may not be rational. e.g. $\frac{\pi}{4}$, $\frac{1}{\sqrt{2}}$ Note: A fraction with irrational numbers in the numerator *and* denominator may become rational. e.g. $\frac{\sqrt{2}}{\sqrt{8}}$.

AN "OUT OF THE ORDINARY" QUESTION

V. Lakshminarayanan, Math Instructor, Valasaravakkam, Chennai

The following interesting question was found in an entrance test recently. Since the question and the solution both are nice, they are being shared here.

Question:

If $x+\dfrac{1}{x}=k$, *find* $x^{13}+\dfrac{1}{x^{13}}$.

Generalize further and use the same to verify the answer you found.

Solution:

It is easy to find $x^2+\dfrac{1}{x^2}$, when $x+\dfrac{1}{x}=k$.

$$x^2+\frac{1}{x^2}=(x+\frac{1}{x})^2-2=k^2-2. \quad \text{.......................................(1)}$$

Similarly, $$x^3+\frac{1}{x^3}=(x+\frac{1}{x})^3-3(x+\frac{1}{x})=k^3-3k \text{........................(2)}$$

By (1), $$x^4+\frac{1}{x^4}=(x^2+\frac{1}{x^2})^2-2=(k^2-2)^2-2 \quad \text{..........................(3)}$$

By (2), $$x^6+\frac{1}{x^6}=(x^3+\frac{1}{x^3})^2-2=(k^3-3k)^2-2 \quad \text{..........................(4)}$$

$$x^7+\frac{1}{x^7}=(x^4+\frac{1}{x^4})(x^3+\frac{1}{x^3})-(x+\frac{1}{x})$$

By (2)&(3), $$=[(k^2-2)^2-2][k^3-3k]-k$$

$$=(k^4-4k^2+2)(k^3-3k)-k \quad \text{.............................(5)}$$

Finally, $$x^{13}+\frac{1}{x^{13}}=(x^7+\frac{1}{x^7})(x^6+\frac{1}{x^6})-(x+\frac{1}{x})$$

$$=[(k^4-4k^2+2)(k^3-3k)-k][(k^3-3k)^2-2]-k$$

which is the required answer.

Now let us try to generalize the process.

Observe that

$$(x^p + \frac{1}{x^p})(x^q + \frac{1}{x^q}) = x^{p+q} + x^{p-q} + x^{q-p} + \frac{1}{x^{p+q}}$$

$$= (x^{p+q} + \frac{1}{x^{p+q}}) + (x^{p-q} + \frac{1}{x^{p-q}}) \quad \text{..........................(6)}$$

--------(6)

From (6), $(x^{p+q} + \frac{1}{x^{p+q}}) = (x^p + \frac{1}{x^p})(x^q + \frac{1}{x^q}) - (x^{p-q} + \frac{1}{x^{p-q}})$

....(7)

Formula (7) is useful to reduce the evaluation of $(x^{p+q} + \frac{1}{x^{p+q}})$, step by step, since it is expressed in the same form but with lesser powers.
We can verify this use of (7) in our case:

$p = q = 1 \ \Rightarrow x^2 + \frac{1}{x^2}. = (x + \frac{1}{x})(x + \frac{1}{x}) - (x^0 + \frac{1}{x^0}) = (x + \frac{1}{x})^2 - 2 = k^2 - 2$

-------------------------------(1)

$p = q = 2 \Rightarrow x^4 + \frac{1}{x^4}. = (x^2 + \frac{1}{x^2})^2 - 2 = (k^2 - 2)^2 - 2 = k^4 - 4k^2 + 2$

-------------------------------(3)

$p = 1,\ q = 2 \Rightarrow x^3 + \frac{1}{x^3} = (x + \frac{1}{x})(x^2 + \frac{1}{x^2} - 1) = k(k^2 - 3)$ -----------------------(2)

$p = q = 3 \Rightarrow x^6 + \frac{1}{x^6} = (x^3 + \frac{1}{x^3})^2 - 2 = (k^3 - 3k)^2 - 2 = a^6 - 6a^4 + 9a^2 - 2$ ---------(4)

and we can proceed as before to get the final answer.

An easy fun!

Take any number like 29. When you multiply it by 3, you get 87.
Find the sum:
87+ its predecessor + the predecessor of the predecessor = 87+86+85= 258
Find the digit sum: 2 +5 + 8 = 15; repeat till you get a 1-digit number: 1+5 =6.
Try this with many other numbers. Do you always get 6? Why?
It should be easy for JMs to find out the reason.

WHAT IS WRONG

1. A student wanted to subtract 45 from 45. He did as follows:

$$\begin{array}{rcl}
\mathbf{45} & = & 9+8+7+6+5+4+3+2+1 \\
\underline{\mathbf{45}} & = & \underline{1+2+3+4+5+6+7+8+9} \\
\mathbf{45 - 45} & = & 8+6+4+1+9+7+5+3+2 \\
 & = & \mathbf{45.}
\end{array}$$

Starting from right to left, how was the subtraction done?
One cannot subtract 9 from 1. So borrow 10 from the previous place 2 and now (10+1) – 9 = 2.
The second place from the right has digit 1 (instead of 2, since a ten has been already taken away to subtract 9 from 1).
Now 8 cannot be subtracted from 1 and so you get a 10 from the previous place occupied by 3 so that now you can get (10+1) – 8 = 3.
The same procedure is followed at every stage and the final answer is

$$8+6+4+1+9+7+5+3+2 = \mathbf{45.}$$

How is this possible?

---

2. Start with

$$1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 1 \qquad \text{...............(1)}$$

$$1 + \sin 45^o - \cos 45^o = \tan 45 \qquad \text{...............(2)}$$

$$1 + \sin 45^o - \cos 45^o = \frac{\sin 45^o}{\cos 45^o} \qquad \text{...............................(3)}$$

Multiplying throughout by $\cos 45^o$,

$$\cos 45^o + \sin 45^o.\cos 45^o - \cos^2 45^o = \sin 45^o$$

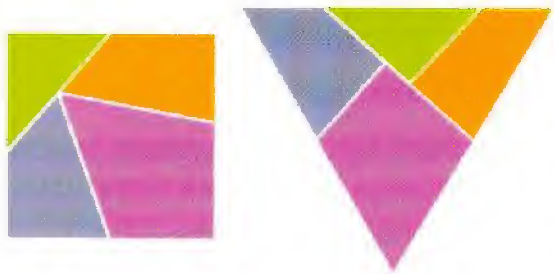
$$\Rightarrow \quad \cos 45^o(\sin 45^o - \cos 45^o) = (\sin 45^o - \cos 45^o) \text{..............(4)}$$

$$\Rightarrow \quad \cos 45^o = 1.$$

Is the answer ok?

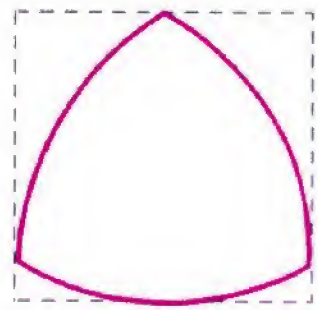
1. The error lies in the stage that the borrowed unit was given the status of ten.
2. The final result got from (4) is due to the error that we cancelled (that is divided both sides) of (4) by an expression equal to zero.

DO YOU KNOW ?



There is a theorem known as Wallace–Bolyai–Gerwien theorem which says that a square can be cut into parts and rearranged into a triangle of equal area.

CURVE OF CONSTANT WIDTH



A curve of constant width is a convex curve whose width (that is, the distance between parallel "supporting lines" bounding it is the same. Such curves when rotated in a square, make contact with all the four sides. All curves of constant width w have the same perimeter πw.

The width of a circle is constant: its diameter. There are non-circular examples too. Reuleaux triangle is one such curve (shown here). To construct a Reuleaux triangle, start with an equilateral triangle and then replace each side by a circular arc with the other two original sides as radii. Reuleaux triangle is named after German engineer Franz Reuleaux.

Guan Baihua, 50, a retired Chinese military officer in Qingdao, has conceived this Reuleaux-Wheeled Bicycle. The front wheel is a pentagonal curve of constant width while the back wheel is a reuleaux triangle (a triangular curve of constant width).

FOLK MATHEMATICS IN NORTH TAMILNADU: A SAMPLE

Even though the world got introduced to the computer technology in the late forties, India bought its first computer (called HEC-2M) only in 1956 for a princely sum of Rs 10 lakhs and installed it at Kolkata's Indian Statistical Institute. It was a time when people necessarily had to remember concepts thoroughly and had to be experts in mental manipulative skills. Children for their part, used to play a number of indoor and outdoor games, involving physical activity and also use of simple mathematics.

"Marriage of rats" was a popular song-based game played by them in which learning of cardinal numbers took placetactfully. The reader will find the song and its translation hereunder:

அப்பா குட்டி மகன் சுப்பாக் குட்டி
சுப்பா குட்டி மகன் சூரியக் குட்டி
சூர்யக் குட்டி மகன் சுண்டெலிராஜா
சுண்டெலி ராஜாவுக்குக் கல்யாணமாம்.

ஒரு எலி ஓடி வந்து ஒரு கால் நட்டதாம்
ரெண்டு எலி ஓடி வந்து ரெண்டு கால் நட்டதாம்
மூணு எலி ஓடி வந்து முக்காலி போட்டதாம்
நாலு எலி ஓடி வந்து நாற்காலி போட்டதாம்
அஞ்சு எலி ஓடி வந்து மஞ்சள் அரைச்சதாம்

ஆறு எலி ஓடி வந்து ஆலம் கலக்கிறதாம்
ஏழு எலி ஓடி வந்து எக்காளம் போட்டதாம்
எட்டு எலி ஓடி வந்து எட்டி எட்டிப் பார்த்ததாம்
ஒன்பது எலி ஓடி வந்து ஓமம் வளர்த்ததாம்
பத்து எலி ஓடி வந்து பந்தல் போட்டதாம்.

Appa Kutti's son is Suppakkutti

Suppa Kutti's son is Suryakutti

Surya Kutti's son is the Ruler of little rats

Rejoice the marriage of this Ruler of little rats.

One rat came running, planted one pillar

Two rats came running, planted two pillars.

Three rats came running, placed the tripod seats

Four rats came running, placed a four legged chair

Five rats came running, ground the turmeric

Six rats came running, mixed the *Aarti*

Seven rats came running, made a lot of noise.

Eight rats came running, peeped and observed

Nine rats came running, stirred up the sacred fire

Ten rats came running, erected the *pandal.*

How nice it would be to have fun through such musical games which also indirectly teach mathematics. If readers come across such songs of 'folk mathematics' (in any Indian language), they can send them to JM; they could be published if foundappropriate.

[Ref: Subramanian S.V. *Tamil Folklore Studies*. Pub: International Insititute of Tamil Studies (1979)]

Amazing pairs! (compare the products and sums)

Pair of numbers	Their sum	Their product
9 and 9	18	81
3 and 24	27	72
2 and 47	49	94
2 and 497	499	994

Are there more of this kind? Investigate.

A NOTE ON PYTHAGOREAN TRIPLETS

Lokesh, Class VIII, T I Matric Higher Secondary School, Ambattur, Chennai – 53

When my math teacher introduced Pythagoras Theorem and Pythagorean Triplet, I got interested to know more on Pythagorean triplets. I started working on it. My teacher gave me three basic triplets like (3,4,5), (5,12,13) and (7,24,25). My investigation led me to the following findings:

Any basic triplet multiplied by a constant

I was thrilled to find that (6,8,10), (9,12,15), (12,16,20),...also fell into the group. i.e. any triplet got by multiplying the basic triplet by a constant is also Pythagorean.

Patterns observed in Pythagorean Triplets

A		
3	4	5
	+ 8	
5	12	13
	+12	
7	24	25
	+16	
9	40	41
	+20	
11	60	61
	+24	
3	84	85

B			
Odd numbers From 3	3	(3x1) + 1	[(3x1) + 1] + 1
	5	(5x2) + 2	[(5x2) + 2] + 1
	7	(7x3) + 3	[(7x3) + 3] + 1
	9	(9 x4) + 4	[(9 x4) + 4] + 1

2n – 1	*2n – 1) (n – 1) + (n – 1)*	*(2n – 1) (n – 1) + (n – 1) + 1*
2n – 1	***2n(n – 1)***	***2n(n – 1) + 1***

$\therefore$ the general formula is $n = 2,3,4\ldots$ It works for other rational numbers too.

Let $n = \frac{4}{3}$ $\quad 2n - 1 = \frac{5}{3}$ $\quad 2n\,(n-1) = \frac{8}{9}$ $\quad 2n(n-1) + 1 = \frac{17}{9}$

But if $n = \frac{1}{4}$, $\quad 2n - 1 = -\frac{1}{2}$. $\quad$ So, $n > \frac{1}{2}$

			SUM
3	4	5	$12 = 3 \times 4$
5	12	13	$30 = 5 \times 6$
7	24	25	$56 = 7 \times 8$
$\frac{5}{3}$	$\frac{8}{9}$	$\frac{17}{9}$	$\frac{40}{9} = \frac{5}{3} \times \frac{8}{3} \ldots$

In general, $\quad$ $\mathbf{2n - 1}$ $\quad$ $\mathbf{2n\,(n - 1)}$ $\quad$ $\mathbf{2n(n - 1) + 1}$ $\quad$ $\mathbf{(2n - 1)\,2n}$

How do you add?

Suppose you want to find 737 + 623 + 486 + 336.

Method 1

	1	2	
	7	3	7
	6	2	3
	4	8	6
	3	3	6
			2
		8	
2	1		
2	1	8	2

Method 2

	7	3	7
	6	2	3
	4	8	6
	3	3	6
		2	2
	1	6	
2	0		
2	1	8	2

Method 3

	7	3	7
	6	2	3
	4	8	6
	3	3	6
2	0	0	0
	1	6	0
		2	2
2	1	8	2

Method 1 is what we usually do in the classroom.

In Method 2, we start with the ones column, add each column separately amd move the sum of each column one space to the left; then find the total.

In Method 3, we find the sum of the left-hand column of addends and annex zeros to correspond with place values.

Are there similar procedures in subtraction too?

Have you seen this ?

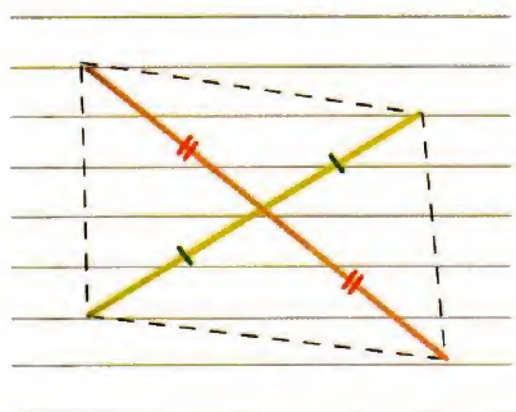
The pantograph is a simple apparatus for copying drawings, maps, designs, etc., on a reduced or enlarged scale, or to the same size as the original. Try to learn the mathe-matics behind it.

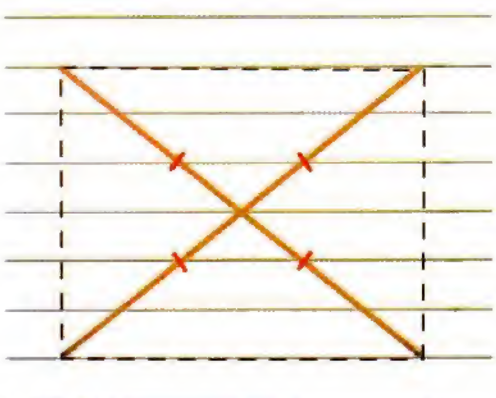
SIMPLE GEOMETRICAL EXPLORTIONS

*Dr. M.*Palanivasan. HM, I.C.F.*Aided Hr.Sec.School, Chennai-600 038*

A few ruled sheets and some pieces of broom-sticks are sufficient to learn a few nice ideas in Geometry. That is what we did during our Project Day. I am sharing my experiences here.

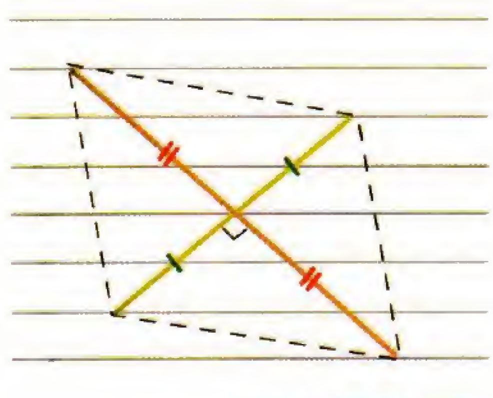
1. Take two **unequal** broom stickpieces whose lengths are known.
Place them such that their mid points coincide (as shown).
What figure do you get if you join the four corners?
Why and how?

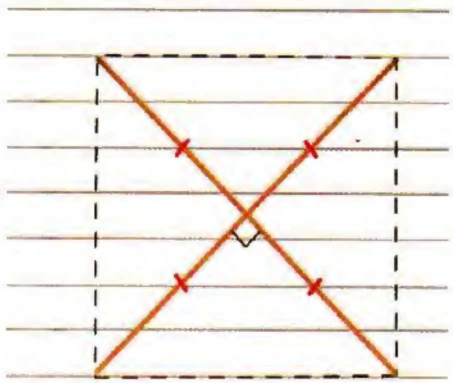




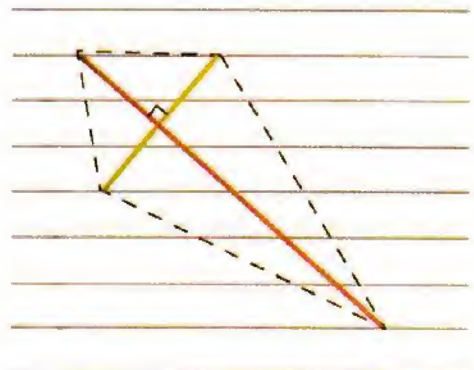
2. Take two **equal** broom stick pieces whose lengths are known. Place them such that their mid points coincide (as shown).What figure do you get if you join the four corners? Why and how?

3. Take two **unequal** broom stick pieces whose lengths are known. Place them intersecting perpendicularly such that their mid points coincide. What figure do you get if you join the four corners?
Why and how?

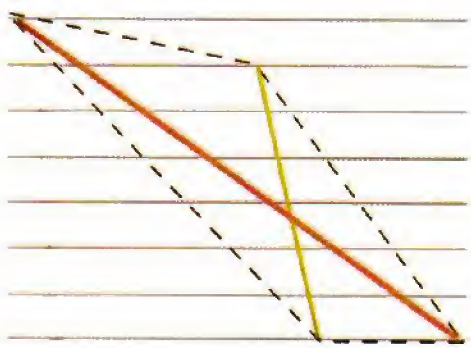




4. Take two **equal** broom stick pieces whose lengths are known. Place them intersecting perpendicularly such that their mid points coincide. What figure do you get if you join the four corners? Why and how?



5. Take two **unequal** broom stick pieces. Among the two pieces, one piece passes through the midpoint of the other perpendicularly. What figure do you get if you join the four corners? Why and how?



6. Take two **unequal** broom stick pieces whose mid points are known. Place them intersecting each other at a point which is not the midpoint of either piece. What figure do you get if you join the four corners? Why and how?

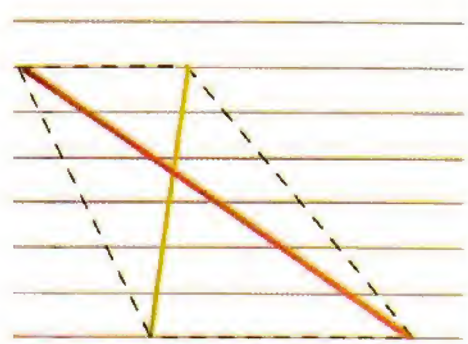
7. Take two **unequal** broom stick pieces whose midpoints are known. Place them intersecting such that

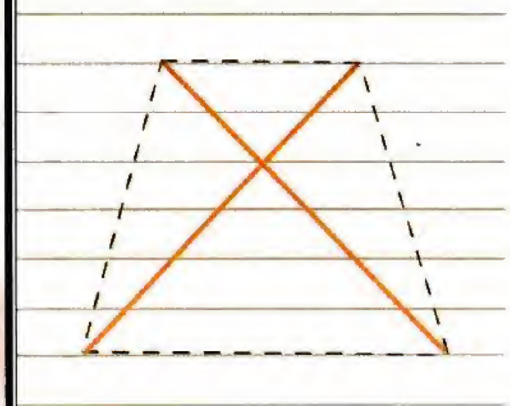
(i) their top points are on the same ruling,

(ii) their bottom points are on the same ruling and

(iii) the intersecting point is not the midpoint of either piece.

What figure do you get if you join the four corners? Why and how?





8. Take two **equal** broom stick pieces whose mid points are known. Place them intersecting such that
i) their top points are on the same ruling;
ii) their bottom points are on the same ruling.
The pieces need not cut the midpoint of either piece.

What figure do you get if you join the four corners? Why and how?

What we did during the day was all about various special quadrilaterals. The different questions were based on the properties of the diagonals of such quadrilaterals.

The shapes discussed were easy to guess. Anyhow they are given here for the sake of completeness. It is up to the readers to discuss and find out why these are the correct answers.

Answers:

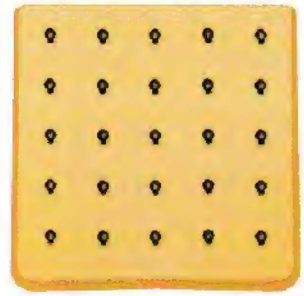
1) Parallelogram	2) Rectangle	3) Rhombus	4) Square
5) Kite	6) Quadrilateral	7) Trapezium	8) Isosceles trapezium

Shapes are important!

A boomerang comes back when you throw it. How? Its peculiar path of is due to a combination of its *shape* and the way it's thrown, making them more than just your average projectile.
With an understanding of the aero-dynamics that drive it, skilled throwers can make their boomerangs race for hundreds of metres.

A WONDERFUL LEARNING TOOL: THE GEOBOARD

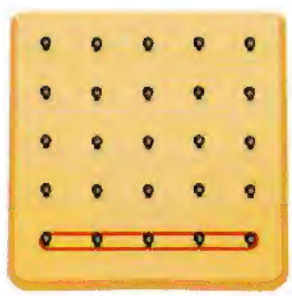
R. Nandhini, Math Dept., Sri Sarada Secondary School, Chennai-600 086.

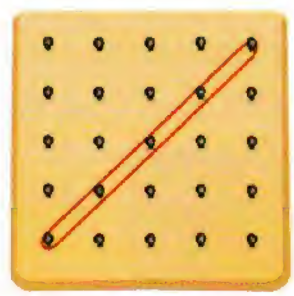


The writer wishes to share with the readers the well-known concept of Geoboard and activities using it to learn/ experiment with Plane Geometrical concepts. This is just to stress upon its usefulness in the class room and also at home for doing projects.

Geo-boards are grids of pegs that can hold rubber bands in position. One can make a geo-board at home. Just fit nails or screws at fixed intervals on acrylic or wooden board. It is great to do activities on geo-board with one or more of one's friends. That gives scope for a lot of mathematical communication.

Making shapes presents opportunities to discover the properties of the shapes. It is assumed that the distance between two adjacent horizontal (or vertical nails) is a unit of length. Measuring length is the initial job. Computing the length of a vertical or horizontal line segment is easy; but how about the length of a diagonal line segment? You will need Pythagoras theorem!





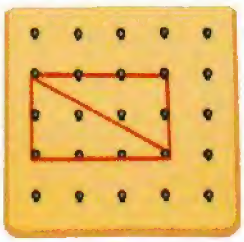
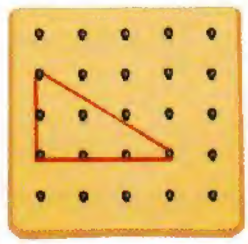
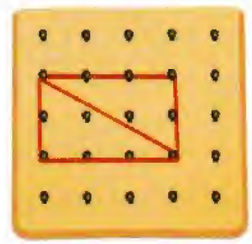
Once you are comfortable with measuring lengths on geoboard, you can experiment with perimeters of polygonal figures. Here are some suggestive explorations:

i. Can two rectangles of different lengths have the same perimeter?
ii. Make a polygon of perimeter 7 whose angles are all right angles.
iii. Is there only one method of making a polygon with perimeter 8?
iv. Is it possible to have two or more different triangles with the same perimeter?

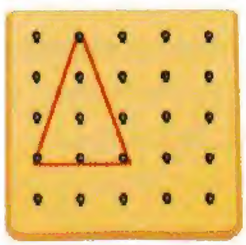
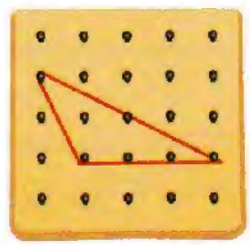
v. Make a square with perimeter 20 units, in two different ways.

vi. Can you construct an equilateral triangle on a geo-board?

The next stage is to find the area of a rectangle and a right triangle constructed with rubber bands on the geo-board. With techniques to find these, it is possible to experiment with the areas of other closed shapes formed by line-segments.

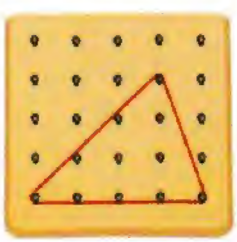
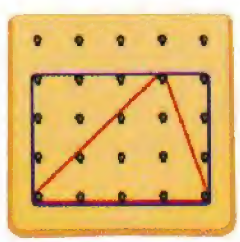
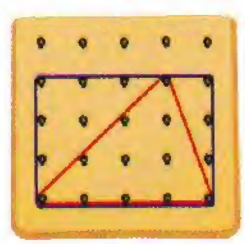




Rectangle (Area = 6) Right triangle = ½ the rectangle

Acute triangle obtuse triangle

It is important to note how "rectangularization" helps to find the areas of triangles.

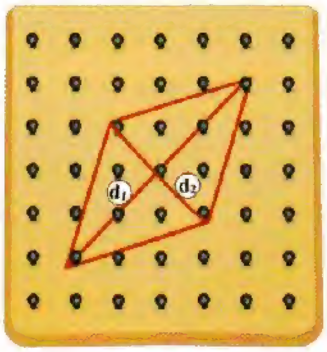




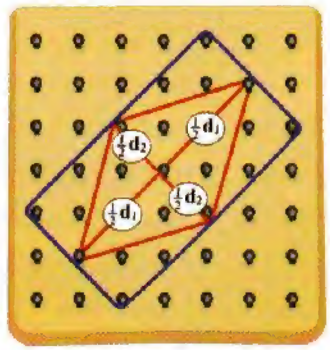
Given triangle Inscribed in a rectangle Triangle = ½ rectangle

Using this technique of surrounding the figure with a rectangle is a very simple technique to get even a general formula.

Take the case of a rhombus, for example.

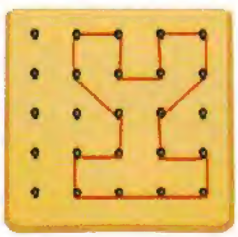
Here is the rhombus, with diagonals of length d_1 and d_2.



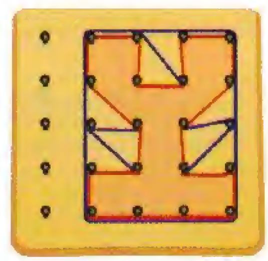


Now you find that the area is = ½ rect.ABCD = $\frac{1}{2}d_1d_2$.

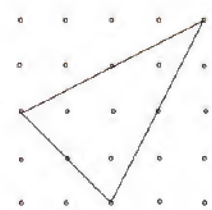
You can use this technique in various problems to calculate areas of rectilinear figures.



Red area = Rectangle minus the Blue triangles.



You can have a lot of problems to investigate for pleasure:



i. In how many ways can you find the areas of the given triangle?

ii. Can you construct a triangle which has half the area of the given triangle?

iii. Can two pentagons have the same area but different perimeter (or vice-versa).

iv. Make a figure that has an area of 4 and a perimeter of 10.

v. Construct any polygon using rubber bands. Divide it into 4 or more unequal parts. Express each part as a fraction of the whole area of the polygon.

vi. How may different size squares can you make on your geo-board? Find the area of each.

vii. Construct any triangle *ABC* and draw one more triangle *PQR* whose sides are simple doubles of the sides of *ABC*. Compare the areas of the two triangles.

viii. Repeat the above for squares and rectangles. What do you find?

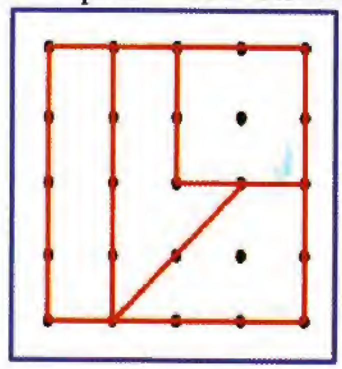
ix. Make 6 different shapes that each have an area of 10 square units

x. How will you construct an equilateral triangle on a geo-board?

xi. Demonstrate two triangles which have the same shape but are different in sizes. What is the area of each?

xii. How many different triangles can be found on a 3 x 3 geo-board? Classify them according to: size of angles, length of sides, lines of symmetry, order of rotational symmetry. Find their areas.

If you do not have a geo-board, you can use a square dot sheet. Observe the following problem:

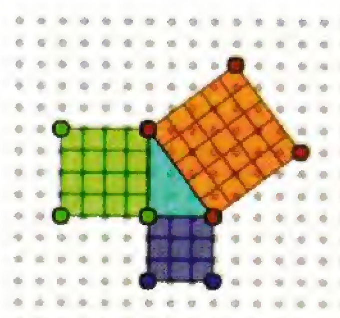


If you have a square geo-board made of 25 nails, how will you divide it into four regions of equal area such that no two figures you obtain are same?

Here is one solution: (This need not be the unique solution; you have a lot of ways to get several varied solutions.

Work out such problems as a group and when you compare your results with one another, you will understand how creative you can be.

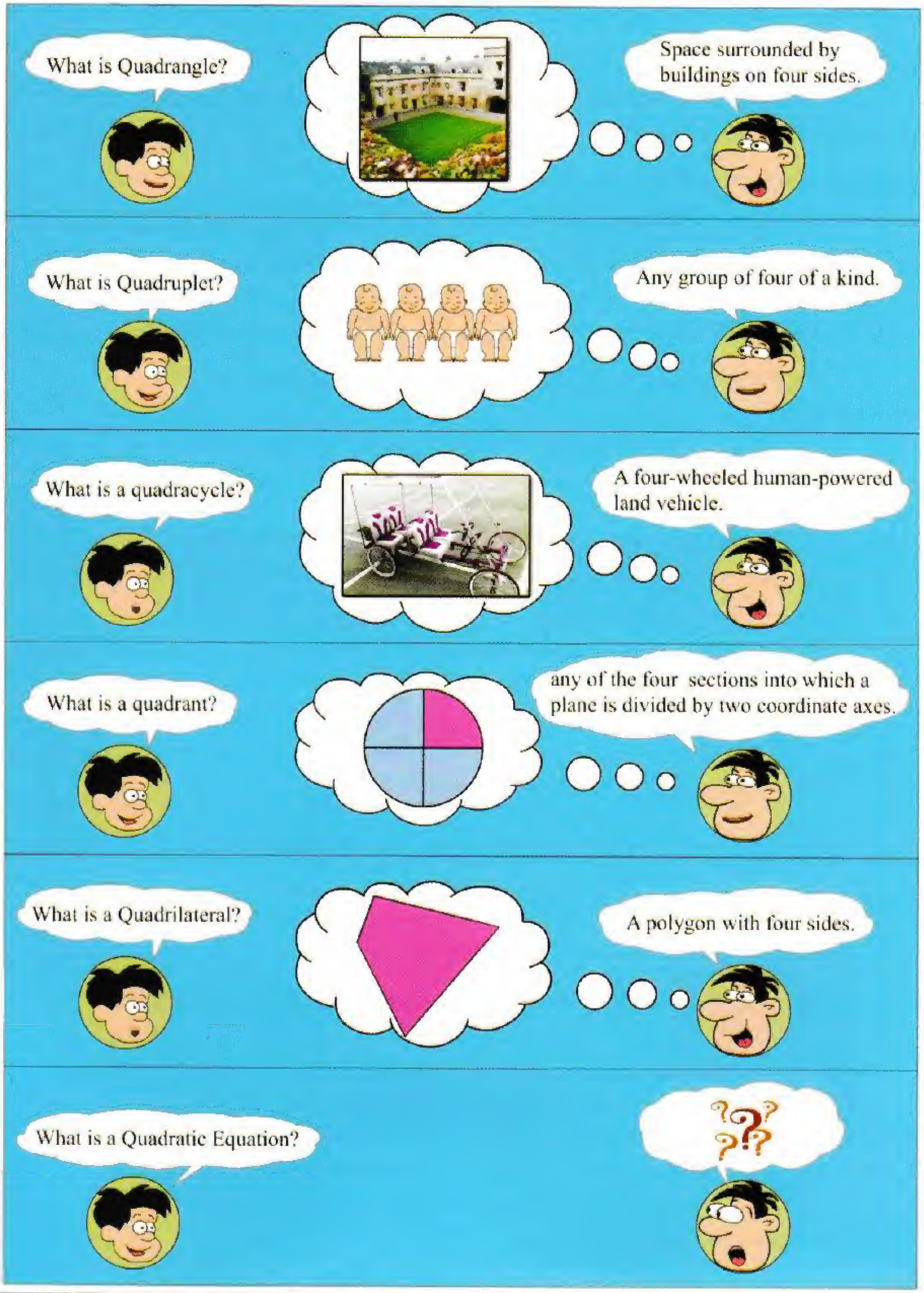
Can you now calculate the area of each individual figure that is formed?



These investigations are a good leisure time activities during the summer vacation. Once begun, they will be more exciting and challenging than computer games! Later on, you can even go to the extent of verifying geometrical theorems.

Make a geo-board just with 25 push-pins, rubber bands, thick hardboard and a hammer. Your Journey to an exciting mathematics can begin there!

WHAT IS IN THE NAME?



TURNING UPSIDE DOWN

Reciprocal of a fraction is just turning the number upside down. (You cannot have a fraction with denominator zero; but you can treat a non-zero whole number like 7 as $\frac{7}{1}$).

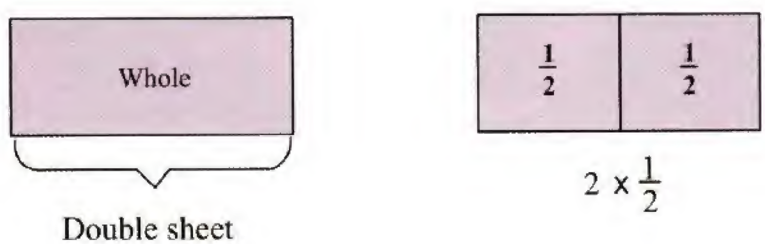
A number, multiplied by its reciprocal is 1. Why? We can get convinced by the following illustrations:

1) Take a double sheet of paper. Let it represent a whole.

Fold it into two halves. Note that 2 times $\frac{1}{2}$ is 1.

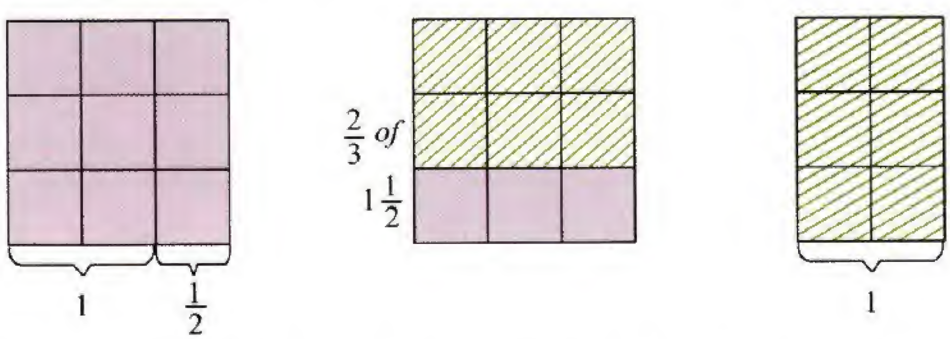
That is, $2 \times \frac{1}{2} = 1$.

A)



2) Take a rectangular sheet to represent $1\frac{1}{2}=\frac{3}{2}$.

Fold it into three equal parts. Two of the equal parts (shaded in the figure) represent $\frac{2}{3} \times 1\frac{1}{2}=1.\ \frac{2}{3}$ *of* $1\frac{1}{2}$.

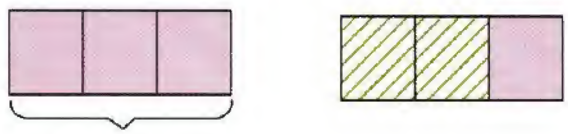


You now find that $\frac{2}{3}$ *of* $1\frac{1}{2}$ is 1. That is, $\frac{2}{3} \times 1\frac{1}{2}=1$.

3) Take 9 squares of a squared sheet to represent $1\frac{1}{2}$. The two top rows represent $\frac{2}{3}$ of this $1\frac{1}{2}$.

Now rearrange the top two rows into a whole.

This shows $1\frac{1}{2} \times \frac{2}{3} = 1$.



Represents

$1\frac{1}{2} = \left(\frac{3}{2}\right)$ $\qquad$ $\left(\frac{2}{3}\right) of = 1\frac{1}{2}$

We can use a little algebra to see this fact generally.

Let $\frac{p}{q}$ be a fraction.

The reciprocal of $\frac{p}{q}$ is $\frac{q}{p}$.

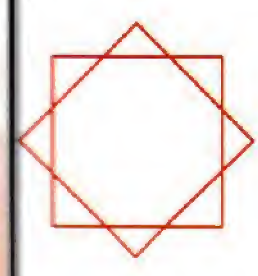
The product of the fraction and its reciprocal is $\frac{p}{q} \times \frac{q}{p}$, which is $\frac{pq}{qp}$.

Since $pq = qp$, we get $\frac{p}{q} \times \frac{q}{p}, = \frac{pq}{qp} = 1$.

What is the reciprocal of zero?

It must be $\frac{1}{0}$, but 0 cannot be in the denominator of a fraction. So you cannot have a reciprocal for zero. $\frac{1}{0}$

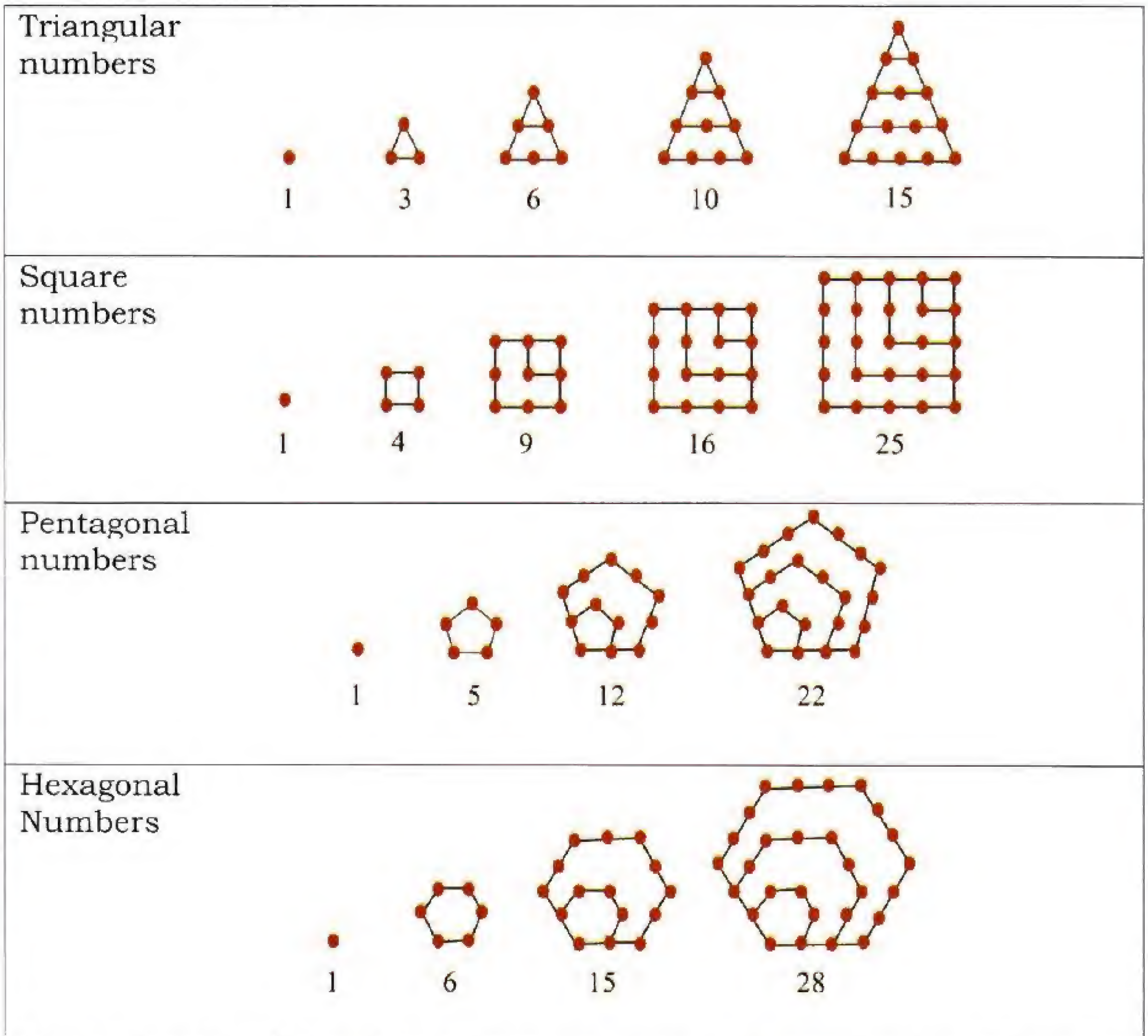
Mathematics in Religion -The Star of Lakshmi



It is an eight-pointed star (a special octagram) made up of two squares and is said to represent the eight types of wealth provided by goddess Ashtalakshmi. The interior of a Star of Lakshmi with edges of length a is a regular octagon with side lengths $(\sqrt{2}-1)a$

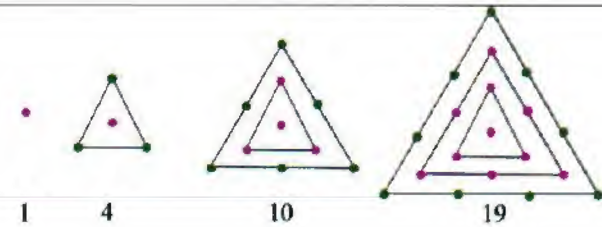
STELLAR NUMBERS

In one of the earlier issues of JM, we have see what Figurate numbers are. Let us recall the ideas. A figurate number is a number that can be symbolized by a regular geometrical arrangement of equally spaced points. If the arrangement forms a regular polygon, the number is called a polygonal number. Here are some examples:



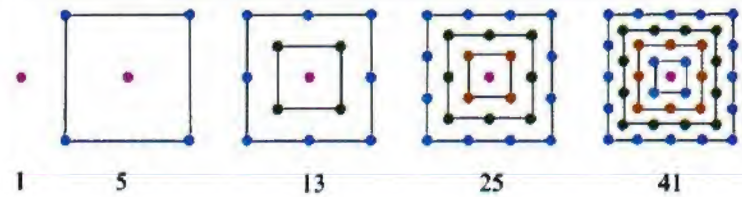
The **centered polygonal numbers** are a special type of series of figurate numbers. In this case, each figure is formed by a central dot, surrounded by polygonal layers with a constant number of sides. Each side of a polygonal layer contains one dot more than a side in the previous layer.Here are some examples:

1. Centred Triangular numbers:



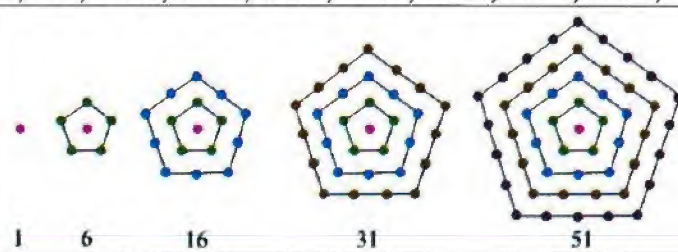
The first few centered triangular numbers are:
1, 4, 10, 19, 31, 46, 64, 85, 109, 136, 166, 199,....etc.

2. Centred Square numbers:



The first few centred square numbers are:
1, 5, 13, 25, 41, 61, 85, 113, 145, 181, 221, 265, 313, 365,.. etc.

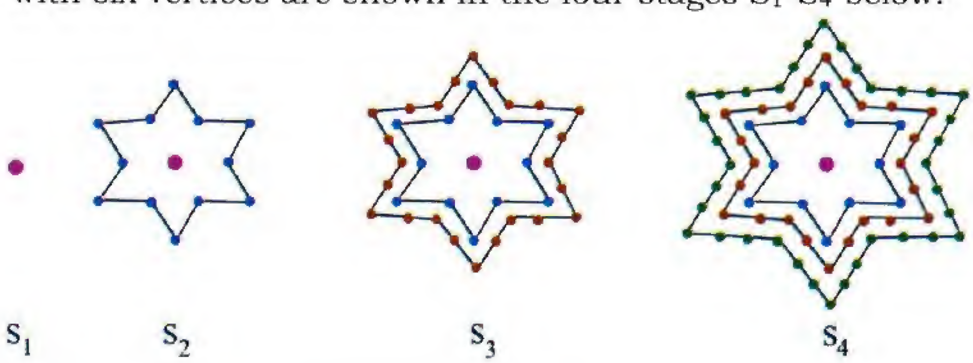
3. Centred Pentagonal numbers:



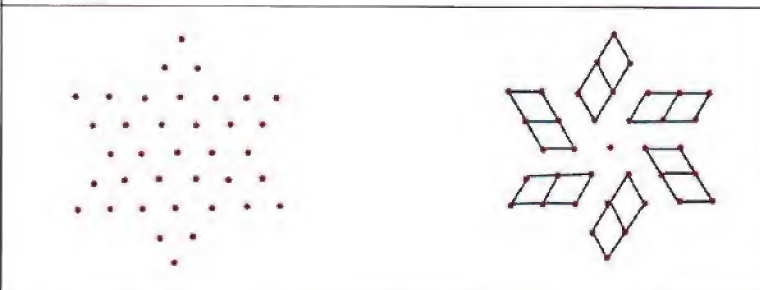
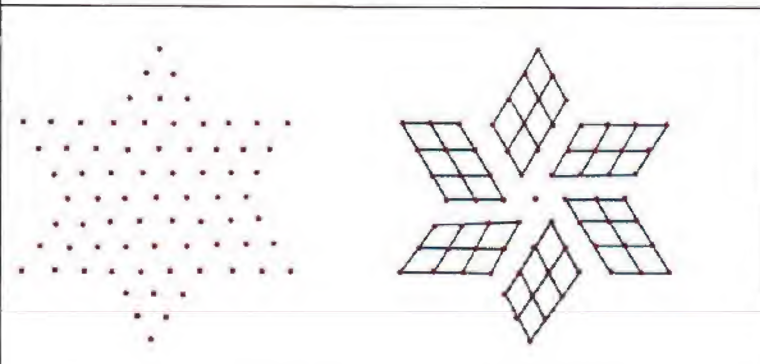
The first few centred Pentagonal numbers are:
1, 6, 16, 31, 51, 76, 106, 141, 181, 226, 276, 331, 391,...etc

We now look into what are called **Stellar Numbers. A stellar number** is a figurate number, based on the number of dots that can fit in a centered hexagram or star shape.

Consider stellar (star) shapes with *p* vertices, leading to p-stellar numbers. The first four representations for a star (a centred hexagon) with six vertices are shown in the four stages S_1-S_4 below.



The 6-stellar number at each stage is the total number of dots in the diagram. Now we find the number of dots (that is the stellar number) in each stage and to organize the data to recognize any possible patterns. Though there are algebraic ways to arrive at a result for number of dots of S_n of any p-stellar, it is relevant to visualize the same (wherever possible) using geometry. We make an attempt now for the 6-stellar:

6-stellar	**Split-diagram**	**No. of dots**
S_1		1
S_2		1 + 6.(2)(2 - 1)
S_3		1 + 6.(3)(3 - 1)
S_4		1 + 6.(4)(4 - 1)

In the sequence of diagrams, we see that the 6-stellars are alternately viewed as sa combination of a centred dot and group of dots joined by the six parallelogram grids. One could visualize that the number of dots in S_n would be

$$1 + 6n(n - 1).$$

The reader is expected to experkiment with other p-stellars so that it can now be seen that the number of dots in S_n of any p-stellar would be

$$1 + pn(n - 1).$$

IDEAS FOR INVESTIGATION BY HIGHER GRADE STUDENTS

(Some of these could be taken up even at Middle School level).

1. What is Modular Arithmetic? Where and how is it useful?

2. Patterns in Pascal's triangle.

3. What are Random numbers? How are they generated? Where are they useful in daily life?

4. Generating Pythagorean triples and their wonders.

5. Mathematics involved in Magic squares.

6. What are Egyptian fractions? How are they interesting and useful?

7. What is special about Euler's Identity?

8. Fascinating Palindromes.

9. Special Numbers like: Perfect Numbers, Cole Numbers, Kaprekar Numbers, Fibonacci Numbers, ... etc.

10. What is a recursive relation? Where do we find them in Maths?

11. The elegance and importance of Euclidean algorithm.

12. Lapses in mathematical reasoning.

13. Mathematical Paradoxes.

14. Some conjectures and their present status.

15. What are Mandelbrot and Julia Sets? Identify some samples of their graphics generated by complex numbers.

16. What is special about Diophantine equations? (Fermat's Last Theorem is one of the most famous such equations).

17. Meaning and uses of Continued fractions. (The great Indian genius Ramanujan discovered some amazing examples of these).

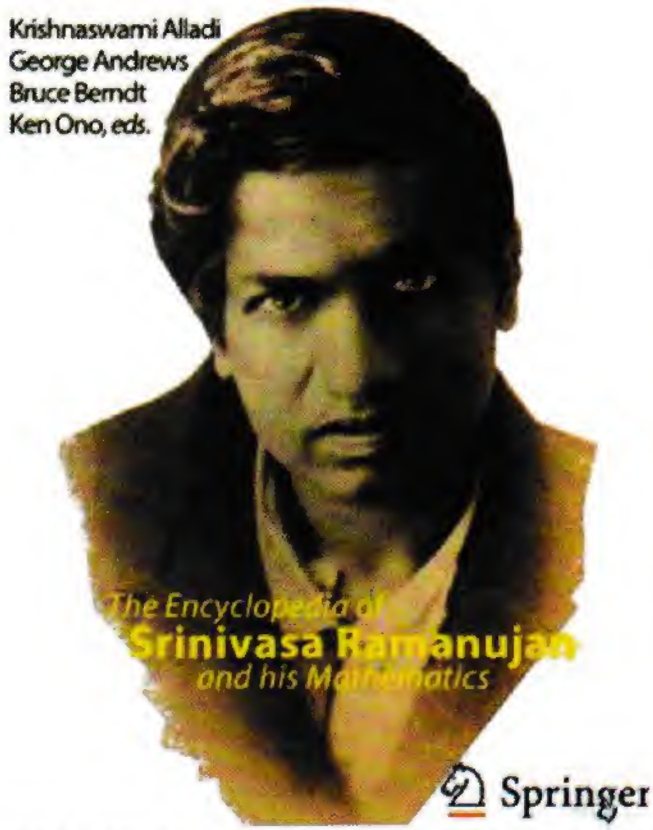
18. Samples of ancient Indian mathematics.

(More to follow in the next issue)

In the News:

RAMANUJAN ENCYCLOPEDIA LAUNCHED

http://www.thehindu.com/sci-tech/science/ramanujan-encyclopedia-launched/article8426583.ece?homepage=true



An encyclopedia of Srinivasa Ramanujan and his mathematics is being launched by Springer.

This was announced by Marc Strauss, Editorial Director, Mathematics, of Springer, North America, at an international conference of mathematics held at the University of Florida.

"The comprehensive encyclopedia of about 1000 pages, in two volumes, will contain everything important about Ramanujan's life and mathematics," explained Mr. Strauss during his announcement at the recently held International Conference on Number Theory at the University of Florida.

"We have assembled a team of leading researchers as Editors-in-Chief, who are experts on Ramanujan, and who have considerable editorial experience, to ensure the success of this massive project," he added. The Editors-in-Chief of the Ramanujan encyclopedia are Professors Krishnaswami Alladi (University of Florida), George Andrews (The Pennsylvania State University), Bruce Berndt (University of Illinois), and Ken Ono (Emory University). "

The Ramanujan Encyclopedia is a comprehensive reference book that will contain information on all the mathematical contributions of Ramanujan and their impact on scientific fields, as well as on important aspects Ramanujan's life including the individuals who have played significant roles in his life and with regard to his work.

The themes that the encyclopedia will include are:

1) Ramanujan's life,
2) persons closely connected with Ramanujan's life or maths.
3) Ramanujan's notebooks and work in India,
4) Ramanujan's letters to Hardy,
5) Ramanujan in England,
6) Ramanujan's published papers,
7) Ramanujan's lost notebook,
8) Ramanujan's work and its influence,
9) Books/expositions on Ramanujan's life and work,
10) Ramanujan in the media,
11) honouring and preserving Ramanujan's legacy,
12) modern developments in research and
13) Ramanujan's health.

"The encyclopedia will contain several hundred entries in the form of articles by experts that will provide a detailed treatment of these themes. But all entries will be listed alphabetically to facilitate easy reference," said Professor Alladi. "The encyclopedia will be updated periodically to keep abreast of current developments," he added.
The Ramanujan encyclopedia is planned not only as a historical document, but also as a valuable reference for those pursuing research on, or related to, Ramanujan's work. It will be of interest to experts and the lay person alike.

"Springer is already playing a big role in the world of Ramanujan with the publication of the edited versions of *Ramanujan's Notebooks* authored by Bruce Berndt, and *Ramanujan's Lost Notebook*authored by George Andrews and Bruce Berndt, as well as *The Ramanujan Journal,* for which Alladi is the Editor-inChief. The Ramanujan encyclopedia is the next big step in Springer's commitment to fostering the legacy of Ramanujan," said Mr. Strauss.

A PECULIAR SET OF SIMULTANEOUS EQUATIONS

V. Swarnarekha, III B.Sc., MOP Vaishnav College, Chennai-600 034.

A pattern of numbers such that the difference between any two consecutive terms is constant, is called an arithmetic progression or arithmetic sequence. For example, the following are arithmetic sequences:

1. $5 \overset{+3}{,} 8 \overset{+3}{,} 11 \overset{+3}{,} 14 \overset{+3}{,} 17, \ldots$ Here terms increase by a common difference 3.
2. 17, 10, 3, -4, -11, -18, ... Here the common difference is -7 (It means the terms decrease by 7).
3. $1, \frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2}, \ldots$ Here the common difference is $-\frac{1}{2}$.

Choose any arithmetic sequence, say,

7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, ...

Choose any six <u>*consecutive*</u> terms, say, 10, 13, 16, 19, 22, 25 and form a pair of simultaneous equations in two variables as follows:

$$10a + 13b = 16 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$19a + 22b = 25 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

What is the solution of this pair?

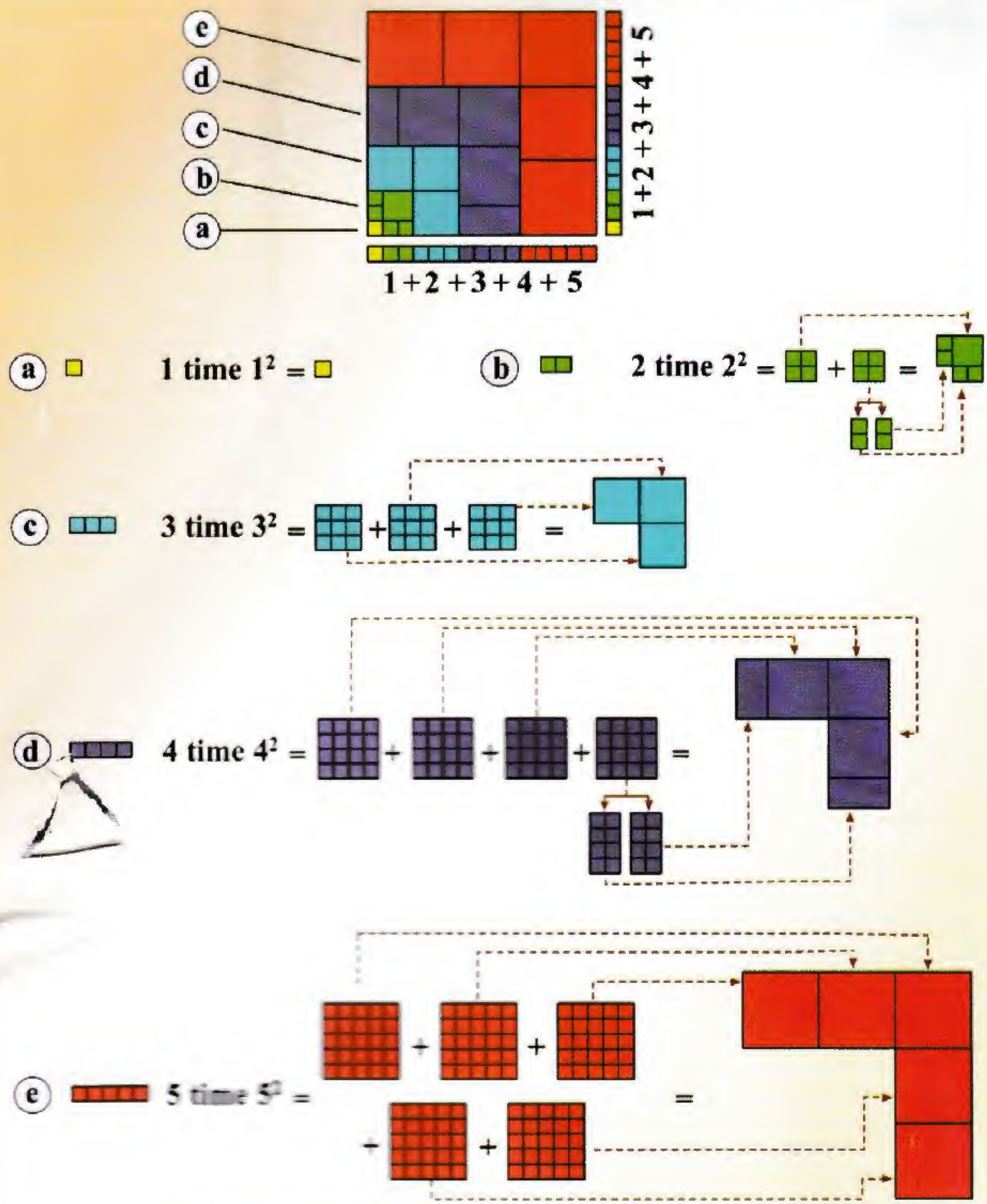
Now choose any other arithmetic sequence, say, -7, -4, -1, 2, 5, 8, 11, 14, ... and form the simultaneous equations by picking up any six consecutive terms,

$$-4a - 1b = 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$5a - 8b = 11 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

What is the solution of this pair?
Repeat this for a few more arithmetic sequences. What do you find? Why?
Is it not something enjoyable?

If you consider a general arithmetic sequence *a, a+d, a+2d, a+3d, ...* and try to solve, the mystery will be smashed. Whenever the first three coefficients come from an arithmetic sequence and sos do the second Three, the solution is always (-1, 2)

A Picture Story



$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2$$

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